# Sparse time-domain multi-dimensional deconvolution for OBN data

G. Poole[1], S. King[1], R. Haacke[1]

[1] CGG

## Summary

Multi-dimensional deconvolution (MDD) is an attractive method to remove free-surface multiples for OBN datasets. Multi-dimensional implementations are able to produce accurate multiple models in areas of structurally complex reflectivity. For most OBN acquisitions, the receiver density is not sufficient for receiver-side MDD. Consequently, source-side methods have been introduced to take advantage of the often denser source sampling. However, source-side MDD is an under-determined inversion problem, which can inhibit the recovery of accurate amplitudes in the multiple model. We introduce a time-domain implementation of source-side MDD, which incorporates sparseness weights to improve the amplitude fidelity of the multiple model. The method uses multi-dimensional annihilation filtering with sparseness weighting, reducing the dimensionality of the least-squares problem and helping the inversion to focus on amplitudes generated by the strongest subsurface reflectors. Application to a North Sea OBN dataset shows an appreciable improvement in amplitude fidelity for MDD multiple models, leading to reduced levels of residual multiple in the data after a subtraction without use of adaptive filtering.

**Sparse time-domain multi-dimensional deconvolution for OBN data**

## Introduction

Seismic data from Ocean Bottom Nodes (OBN) are widely used for subsurface imaging and velocity model building. However, effective multiple attenuation for such data remains a challenge in many areas. When overlying towed-streamer data are available, surface-related multiple elimination (Ikelle, 1999) or wave-equation multiple modelling (Pica et al., 2006) methods may be effective. Up/Down Deconvolution (UDD) is an alternative method for attenuating free-surface multiples (Amundsen, 2001). To improve illumination of the shallow section using mirror migration, UDD was extended to Down/Down Deconvolution (DDD; Hampson and Szumksi, 2020). Implementations of UDD and DDD are widely applied to dip decompositions (e.g., tau-p or FK) of common-receiver gathers, which are necessary to avoid a multi-dimensional summation over the typically sparsely sampled receivers. However, the common-receiver approach can perform poorly where near-surface reflectivity is dipping (Boiero and Bagaini, 2021).

Haacke and Poole (2023) introduced annihilation filtering deconvolution (AFD), which retained the benefits of DDD while having less dependency on accurate knowledge of the direct arrival. This extends naturally into a source-side multi-dimensional deconvolution framework (denoted S-AF-MDD), in which a multi-dimensional operator is found that optimally annihilates the down-going wavefield. In this paper, we extend the S-AF-MDD method to the time-domain and use reflectivity sparseness weights to improve accuracy of the resulting multiple model.

## Method

Based on Haacke and Poole (2023), for each temporal angular frequency, $\omega$, the S-AF-MDD generates direct-arrival data, $\phi$, by summation of the down-going data, $d$, with the annihilation operator, $a$. The annihilation operator takes the form of a spike at time zero and offset zero followed by the reflectivity, which has the effect of attenuating non direct-arrival energy. For receiver location $\boldsymbol{x_r}$ and source with location $\boldsymbol{x_s}$, the summation occurs over a source aperture $\boldsymbol{h}$ specified relative to an aperture centre $\boldsymbol{x_0}$. Summing over shot locations $\boldsymbol{x}'_s$ we have

$$\phi(\boldsymbol{x_r}, \boldsymbol{x_s}, \omega) = \sum_{\boldsymbol{x}'_s=\boldsymbol{x_0}-\boldsymbol{h}}^{\boldsymbol{x_0}+\boldsymbol{h}} d(\boldsymbol{x_r}, \boldsymbol{x}'_s, \omega) a(\boldsymbol{x}'_s, \boldsymbol{x_s}, \omega). \quad (1)$$

The annihilation filter is surface consistent, meaning it may also be convolved with data sharing the same annihilation operator aperture recorded by different receivers. The summation in (1) can be represented by the linear system (with uppercase denoting a matrix)

$$\phi(\boldsymbol{x_r}, \boldsymbol{x_s}, \omega) = D(\boldsymbol{x_r}, \boldsymbol{x_s}', \omega) a(\boldsymbol{x_s}', \boldsymbol{x_s}, \omega), \quad (2)$$

where each row of $D$ contains a common-receiver gather of down-going data for shot points within the annihilation operator aperture. The orthogonality of each temporal frequency means that each frequency may be operated on independently.

For most ocean bottom node (OBN) acquisition geometries, there are typically more shot positions than receiver positions, making the above equation under-determined. We propose to improve the conditioning of the system by formulating the S-AF-MDD equation in the time domain and utilising annihilation operator sparseness weights.

Where operator $F$ is the forward Fourier transform, and $F^{-1}$ is the inverse Fourier transform, a time-domain version (for time $t$) of (2) can be written as

$$\phi(\boldsymbol{x_r}, \boldsymbol{x_s}, t) = F^{-1} D(\boldsymbol{x_r}, \boldsymbol{x_s}', \omega) F a(\boldsymbol{x_s}', \boldsymbol{x_s}, t). \quad (3)$$

While the time-domain formulation does not improve the conditioning of the linear system directly, this can be achieved through the addition of time-domain sparseness weights which reduce the dimensionality of the annihilation operator. Introducing sparseness weights $\psi_a$ for the annihilation operator, as well as weights $\psi_\phi$ acting on the water-wave data, into the linear system we solve for $\hat{a} = {\psi_a}^{-1} a$, where

$$\psi_\phi(\boldsymbol{x_r}, \boldsymbol{x_s}, t)\phi(\boldsymbol{x_r}, \boldsymbol{x_s}, t) = \psi_\phi(\boldsymbol{x_r}, \boldsymbol{x_s}, t) F^{-1} D(\boldsymbol{x_r}, \boldsymbol{x_s}', \omega) F \psi_a(\boldsymbol{x_s}', \boldsymbol{x_s}, t) \hat{a}(\boldsymbol{x_s}', \boldsymbol{x_s}, t). \quad (4)$$

The water-wave domain sparseness weights $\psi_\phi$ may be used to constrain the water-wave prediction to the expected arrival time range, or to mitigate the effect of noise in the data. The annihilation filter sparseness weights $\psi_a$ may be derived using iteratively re-weighted least squares, for example by taking the envelope of the annihilation operator derived in previous iterations (see Trad et al., 2003).

Our implementation solves (4) using the conjugate gradients method, but we have also seen good results from steepest descent. Once the annihilation operator has been found, it may be imaged directly or used for multiple prediction of up-going or down-going data (Haacke and Poole, 2023).

**Results**

The following data example comes from an OBN acquisition on the UK continental shelf. The acquisition involved a receiver array of 50 m × 200 m spacing, with a shot carpet of 25 m × 25 m. The water depth varied between 200 m and 550 m, with most of the seabed dip in the cross-line direction. Hydrophone and vertical geophone measurements were used to generate up-going and down-going wavefields as shown in Figure 1a and Figure 1b, respectively. These produced a UDD reflectivity through common-receiver deconvolution, Figure 1c. The dipping seabed in this area reduces the efficacy of the UDD method, causing residual free-surface multiples to persist in the UDD reflectivity (Figure 1c, black arrow). While the UDD result is naturally output at water-bottom datum, the AFD operator is output at sea-surface datum, providing improved near-angle sampling. All images in Figure 1 have been re-datumed to free-surface datum to make comparisons easier. Here, we may appreciate the improved near-angle sampling of the AFD operator as a slight increase in sharpness at the water-bottom (white arrows). Overall, the AFD exhibits slightly more residual multiple than the UDD result. For display purposes, the spike term naturally present in the AFD output (Haacke and Poole, 2023) has been removed. The result of S-AF-MDD is shown in Figure 1e without the sparsity included in the system. Although S-AF-MDD can overcome the layer-cake assumptions of the common-receiver implementation, in this case, the under-determined nature of the inverse problem has reduced the quality of the result. Figure 1f shows the envelope of the data in Figure 1e, which is used as sparseness on the annihilation operator and produces the sparse S-AF-MDD result shown in Figure 1g. The constraints imposed on the sparse S-AF-MDD result have provided a cleaner result with reduced multiple contamination.

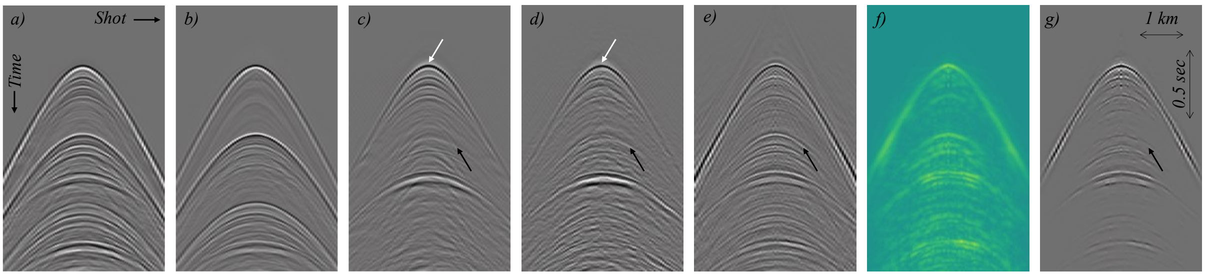


***Figure 1*** *Pre-stack comparison of: a) Up-going receiver gather, b) Down-going receiver gather, c) UDD reflectivity, d) AFD operator, e) non-sparse S-AF-MDD operator, f) S-AF-MDD sparseness weights ($\psi_a$), and g) sparse S-AF-MDD operator. All gathers are re-datumed with shots and receivers at free-surface for easier comparison.*

Stack displays for the down-going input, up-going input, common-receiver UDD result, and common-receiver AFD result are given in Figures 2a-d, respectively. While both the UDD and AFD methods have removed the source signature effectively, we observe residual multiples in both cases (see black arrows), which are caused by the cross-line dip of the seabed in this area violating the layer-cake assumption in the common-receiver implementation. Note the expected improved image sharpness at the water bottom for the AFD result compared to UDD. Figure 2e shows the annihilation filter from S-AF-MDD without sparseness weighting. Compared to Figure 2d, the new result shows an increase in multiple leakage caused by the under-determined nature of the problem. The addition of time-domain sparsity in Figure 2f allows the S-AF-MDD to produce a greatly enhanced result, where the reduced dimensionality of the problem has improved the stability of the inversion. Although the annihilation filter itself may now be too sparse for imaging, it can instead be used for multiple prediction and subtraction.

Figure 3 compares up-going demultiple results and multiple models using multiple predictions from common-receiver UDD (Figures 3a and 3e), common-receiver AFD (Figures 3b and 3f), non-sparse S-AF-MDD (Figures 3c and 3g) and sparse time-domain S-AF-MDD (Figures 3d and 3f). The multiple models are subtracted directly in each case, without adaptive filtering. While residual multiple levels decrease as we move from UDD to AFD and then to non-sparse S-AF-MDD, there is still a small level of residual multiple remaining (arrows). Introduction of time-domain sparseness weights to the S-AF-MDD further reduces the level of residual multiple to almost un-observable levels in the direct subtraction result.

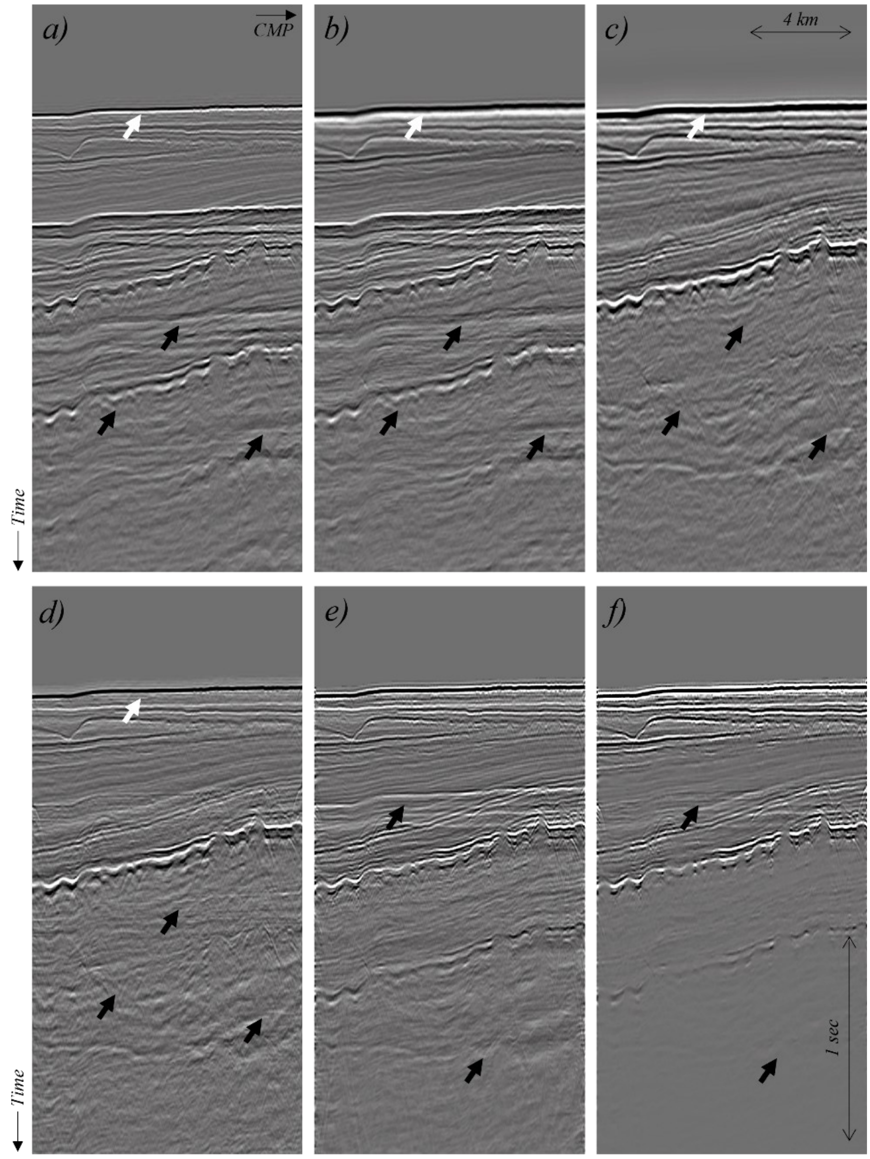


***Figure 2*** *Comparison for a) Down-going mirror-stack, b) Up-going stack, c) common-receiver UDD reflectivity stack, d) common-receiver AFD reflectivity stack, e) non-sparse S-AF-MDD reflectivity stack, and f) time-domain sparse S-AF-MDD reflectivity stack.*

## Conclusions

We have introduced a sparse time-domain implementation of source-side multi-dimensional deconvolution using an annihilation filtering approach. The method includes sparseness weights, which reduce the dimensionality of the annihilation operator. This improves the conditioning of the linear system and the accuracy as well as robustness of the resulting multiple model. The OBN data example shows reduction of residual multiple energy after a model and direct subtraction process, without use of adaptive filtering.

**Acknowledgements**

The authors thank CGG for permission to publish this work, and bp for the data examples.

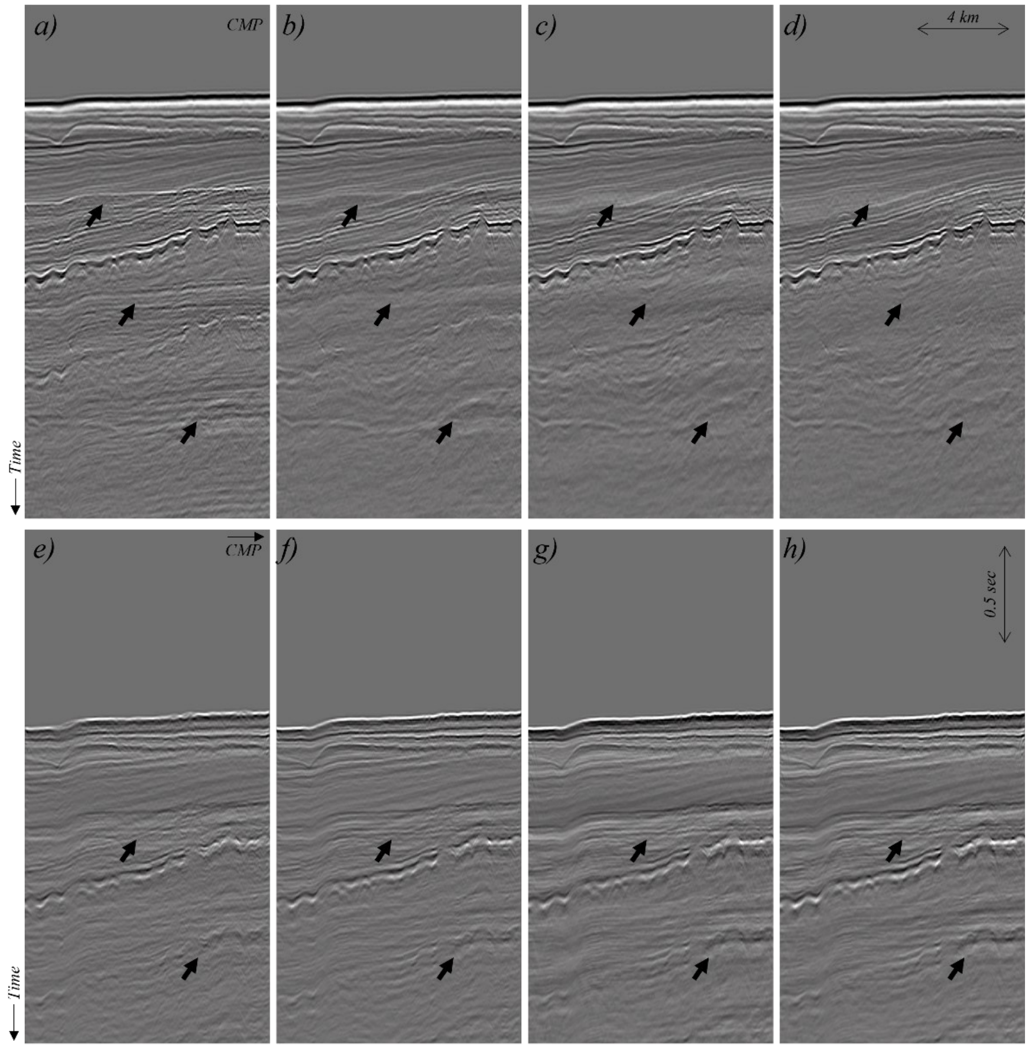


***Figure 3*** *Stack up-going straight subtraction demultiple comparisons using multiple predictions from: a) common-receiver UDD, b) common-receiver AFD, c) non-sparse S-AF-MDD, and d) sparse time domain S-AF-MDD. Multiple models are given in 3e), f), g), and h), respectively. The up-going stack before demultiple is given in Figure 2b.*

**References**

Amundsen, L. [2001] Elimination of free-surface related multiples without the need of the source wavelet. *Geophysics*, **66**(1), 327-341.

Boiero, D. and Bagaini, C. [2021] Up and Down Deconvolution in complex geological scenarios. *Geophysics*, **86**(5), WC55-WC65, doi/10.1190/geo2020-0926.1.

Hampson, G. and Szumksi, G. [2020] Down/down deconvolution. *EAGE Conference & Exhibition*, Extended Abstracts, doi https://doi.org/10.3997/2214-4609.202010603.

Haacke, R. and Poole, G. [2023] Source-side multi-dimensional deconvolution using a downgoing annihilation filter approach. *Second EAGE seabed seismic today workshop*, September 2023.

Ikelle, L. T. [1999] Combining two seismic experiments to attenuate free-surface multiples in OBC data. *Geophysical Prospecting*, **47**(2), 179–193. http://dx.doi.org/10.1046/j.1365-2478.1999.00125.x.

Pica, A., Manin, M., Granger, P.Y., Marin, D., Suaudeau, E., David, B., Poulain, G. and Herrmann, P.H. [2006] 3D SRME on OBN data using waveform multiple modelling. *76th SEG Annual International Meeting,* Expanded Abstracts, doi 10.1190/1.2370074.

Trad, D., Ulrych, T. and Saachi, M. [2003] Latest Views of the Sparse Radon Transform. *Geophysics*, **68**(1), 386-399.